HW PROBLEMS SET 1: ARGUMENT BY CONTRADICTION, INDUCTION

1. Prove that $\sqrt{2}+\sqrt{3}+\sqrt{5}$ is an irrational number.

2. Find the least positive integer n such that any set of n pairwise relatively prime integers greater than 1 and less than 2005 contains at least one prime number.

3. Show that there does not exist a strictly increasing function $f : \mathbb{N} \to \mathbb{N}$ satisfying $f(2) = 3$ and $f(mn) = f(m)f(n)$ for all $m, n \in \mathbb{N}$.

4. Show that the interval $[0, 1]$ cannot be partitioned into two disjoint sets A and B such that $B = A + a$ for some real number a.

5. Consider a collection of N planes in $\mathbb{R}^3$ which all pass through the same point, but no 3 of them intersect at the same line. How many parts do they cut the space into?

6. Prove that for any real numbers x_1, x_2,...,x_n, $n \geq 1$,

$$|\sin x_1| + |\sin x_2| + ... + |\sin x_n| + |\cos(x_1 + x_2 + ... + x_n)| \geq 1.$$

7. Let k be a positive integer. The n-th derivative of $\frac{1}{x^k-1}$ has a form $\frac{P_n(x)}{(x^k-1)^{n+1}}$, where $P_n(x)$ is a polynomial. Find $P_n(1)$.